

## Sterile Neutrinos in $E_6$

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The opportunity to accommodate three flavors of sterile neutrinos exists within the exceptional group  $E_6$ . Implications of this description are discussed.

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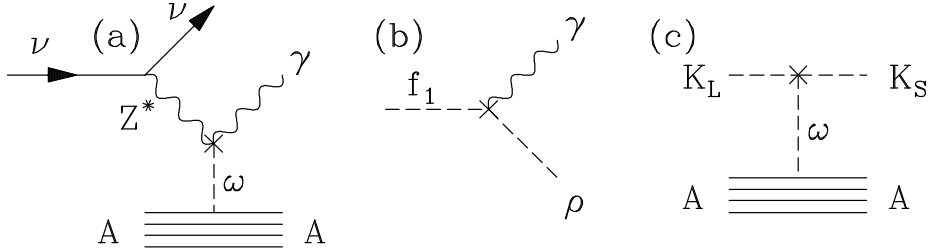
### 1. Introduction

Sterile neutrinos are weak isosinglet neutrinos, visible through mixing with one or more of the three “active” neutrinos  $\nu_e, \nu_\mu, \nu_\tau$ . Several tentative indications exist that the three active neutrinos aren’t enough to fit all oscillation data; sterile neutrinos are one possibility. Present data prefer at least one sterile neutrino, but there are tensions even with two. In the grand unified group  $E_6$  three sterile neutrinos are natural; this talk explores some distinguishing features of such a description.<sup>1</sup>

We first review the shortcomings of a description with only three active neutrinos (Sec. 2); this topic has been covered in greater detail by C. Giunti.<sup>2</sup> We then discuss mass matrices in  $E_6$  and its subgroups (Sec. 3), and their relevance for short-baseline neutrino oscillation experiments (Sec. 4). One of the three sterile neutrinos could play the role of a 7 keV dark matter candidate (Sec. 5). We conclude in Sec. 6.

### 2. Evidence for sterile neutrinos

An early conflict with the picture of three active neutrinos was seen by the LSND experiment at Los Alamos.<sup>3</sup> The MiniBooNE experiment at Fermilab confirmed this result (after a reanalysis of their data).<sup>4–7</sup> The signal is mainly at low energy, falling below an initial energy cut of 475 MeV. It is not clear whether the signal is  $e^\pm$  or photons. A possible photon source would arise from a  $Z-\omega-\gamma$  Wess-Zumino-Witten (WZW) coupling<sup>8,9</sup> giving rise to neutral-current coherent photon production [Fig. 1(a)] off a nuclear target.<sup>10,11</sup>



A virtual  $Z^*$  transforms as a  $J^{PC} = 1^{++}$   $a_1$  meson. The decays  $a_1^0(1260) \rightarrow \omega\gamma$  and the related decay  $a_1^\pm(1260) \rightarrow \rho^\pm\gamma$  are hard to look for. Possible evidence for a related WZW coupling comes from the decay  $f_1(1285) \rightarrow \rho^0\gamma$  [Fig. 1(b)]; the Mark III Collaboration at SPEAR observed this process<sup>12</sup> at a rate 12 times the non-WZW prediction.<sup>13,14</sup> A related term generates an  $\omega K\bar{K}$  coupling giving rise to  $K_S$  regeneration off a nuclear target [Fig. 1(c)]. One estimate of the contribution of the process (a)<sup>15</sup> gives a rate about 1/4 that needed to explain the MiniBooNE result.

A claimed 6% deficit with respect to expectations in the flux of reactor neutrinos could be due to very-short-baseline neutrino oscillations.<sup>16–18</sup> A cautionary note<sup>19</sup> identifies an additional uncertainty associated with 30% of the flux coming from forbidden decays, whose intensity and energy spectra are hard to evaluate.

More evidence in favor of an anomaly comes from the use of  $^{51}\text{Cr}$  and  $^{37}\text{Ar}$  radioactive sources to calibrate the SAGE and Gallex solar neutrino detector,<sup>20,21</sup> finding an observed/predicted ratio of  $0.84 \pm 0.05$ .<sup>22</sup> Both the reactor and the gallium anomalies could be due to short-baseline neutrino oscillations with  $\Delta m^2 = \mathcal{O}(\text{eV}^2)$ . Such a large splitting cannot be accommodated with three active neutrinos, whose masses satisfy  $\Delta m_{21}^2 \simeq 7.6 \times 10^{-5} \text{ eV}^2$ ;  $|\Delta m_{32}^2| \simeq 2.4 \times 10^{-3} \text{ eV}^2$ . A full set of constraints, including ones from the BNL E-776, CDHSW, Daya Bay, ICARUS, KARMEN, MiniBooNE, MINOS, NOMAD, OPERA, SciBooNE, and T2K experiments, is discussed in Ref.<sup>23</sup> and by C. Giunti in this Conference.<sup>2</sup> The absence of oscillations to one flavor ( $N = 1$ ) of sterile neutrino is disfavored at the  $6.3\sigma$  level.

Even with more than one sterile neutrino a basic tension remains between disappearance and appearance experiments. In one fit<sup>24</sup> these are compatible only at the level of 0.008%, mainly owing to a poor fit to the low-energy MiniBooNE “ $e^\pm$ ” signal. For another fit, see Ref.<sup>25</sup> An initial incompatibility between neutrino and antineutrino fits, favoring  $N > 1$ , has been resolved with subsequent data, so there is no longer preference for more than one flavor of sterile neutrino.<sup>2</sup> Nevertheless, should such a need arise in the future, the  $E_6$  scheme provides a natural home for three sterile flavors ( $N = 3$ ).

### 3. Mass matrices in $E_6$ and subgroups

The group  $SU(5)$ <sup>26</sup> is the unique one of rank 4 containing the standard model (SM) group  $SU(3)_{\text{color}} \times SU(2)_L \times U(1)$ . The quarks and leptons belong to  $5^* + 10$

representations; there is no need for a right-handed neutrino. The group  $SO(10)^{27}$  contains  $SU(5)$ ; its 16-plet spinor contains the  $SU(5)$  representations  $5^* + 10 + 1$ , where the  $SU(5)$  singlet is a right-handed neutrino  $N$ . If this state is given a large Majorana mass, the corresponding left-handed neutrino Majorana mass can be made very small (the *seesaw* mechanism<sup>28</sup>). The rank-5 nature of  $SO(10)$  implies the existence of an extra  $U(1)$  and the possible observability of a  $Z'$  at the TeV scale or above.

The exceptional group  $E_6$ <sup>29</sup> contains  $SO(10)$ . Each fundamental 27-plet of  $E_6$  contains  $16 + 10 + 1$  of  $SO(10)$ . The 10 of  $SO(10)$  contains  $5 + 5^*$  of  $SU(5)$ , where the 5 contains a color-triplet weak isosinglet quark and a color-singlet weak isodoublet lepton. The singlet of  $SO(10)$  (“ $n$ ”) is a sterile neutrino candidate. Since one needs three 27’s to account for three families of ordinary quarks and leptons, there are three sterile neutrinos in  $E_6$ . The rank-6 nature of  $E_6$  implies the possibility of two extra  $U(1)$ ’s or at least one linear combination surviving symmetry breaking down to LHC energies. The  $U(1)$  charges are defined as

$$E_6 \rightarrow SO(10) \times U(1)_\psi (Q_\psi) ; \quad SO(10) \rightarrow SU(5) \times U(1)_\chi (Q_\chi) .$$

A  $Z_\theta$  can couple to  $Q_\psi \cos \theta + Q_\chi \sin \theta$ . The combination  $Q_N \equiv -(1/4)Q_\chi + (\sqrt{15}/4)Q_\psi$  vanishes for the right-handed neutrino  $N$ . A large Majorana  $N$  mass is then permitted by  $Q_N$  conservation, enabling a seesaw mechanism with fermion masses generated by a 27-plet Higgs representation.<sup>30</sup> The  $U(1)$  charges for various members of a 27-plet are shown in Table 1.

Table 1.  $U(1)$  charges of 27-plet fermions in  $E_6$ .

27 member ( $SO(10), SU(5)$ )	$2\sqrt{6} Q_\psi$	$2\sqrt{10} Q_\chi$	$2\sqrt{10} Q_N$
$\nu_e(16, 5^*)$	-1	3	-2
$(16, 10)$	-1	-1	-1
$N_e^c(16, 1)$	-1	-5	0
$\nu_E(10, 5^*)$	2	-2	3
$N_E^c(10, 5)$	2	2	2
$n(1, 1)$	-4	0	-5

The  $Z_N$ , coupling to  $Q_N$ , has characteristic branching fractions. Within a single family, 25% of its decays are to ordinary fermions (above the middle line in Table 2) while 75% are to exotic fermions (below the middle line). These consist of a vector-like charged lepton  $E$ , its neutrino  $\nu_E$ , an isosinglet quark  $h$ , and the sterile neutrino  $n$ . If the  $Z_N$  is found at the LHC, it is a potential source of exotic quarks and leptons. The differences between left-handed (L) and right-handed (R) couplings give rise to characteristic production and decay asymmetries.<sup>31</sup>

While  $27 \times 27 = 27^* + 351 + 351'$ , we wish to see what follows from assuming  $27^*$

Table 2. Branching fractions of  $Z_N$  within a family.

Decay product	Helicity		Sum	Percent of total
	L	R		
$e\bar{e}$	4/120	1/120	5/120	4.17
$\nu_e\bar{\nu}_e$	4/120	–	4/120	3.33
$u\bar{u}$	3/120	3/120	6/120	5.00
$d\bar{d}$	3/120	12/120	15/120	12.50
$E\bar{E}$	9/120	4/120	13/120	10.83
$\nu_E\bar{\nu}_E$	9/120	4/120	13/120	10.83
$h\bar{h}$	27/120	12/120	39/120	32.50
$n\bar{n}$	25/120	–	25/120	20.83

dominates, which was a popular assumption in the early days of string theory.<sup>32–36</sup> Some mass matrix elements will be absent as their  $(Q_\psi, Q_\chi)$  values aren't in  $27^*$ . The U(1) charges for the product  $27 \times 27$  are shown in Table 3, where we have listed values of  $(2\sqrt{6} Q_\psi, 2\sqrt{10} Q_\chi, 2\sqrt{10} Q_N)$ . Blank entries denote charges not found in a  $27^*$ -plet, implying a zero entry in the mass matrix. The exception (in the box) is a Majorana mass for the right-handed neutrino  $N_e^c$ , which must be generated by a higher-dimension operator conserving  $Q_N$ .

Table 3. U(1) charges (see text) in the product of two  $27$ 's of  $E_6$ .

	$\nu_e(-1,3,2)$	$N_e^c(-1,-5,0)$	$\nu_E(2,-2,3)$	$N_E^c(2,2,2)$	$n(-4,0,-5)$
$\nu_e(-1,3,2)$	–	$(-2,-2,-2)$	–	$(1,5,0)$	–
$N_e^c(-1,-5,0)$	$(-2,-2,-2)$	$(-,0)$	–	$(1,-3,2)$	–
$\nu_E(2,-2,3)$	–	–	–	$(4,0,5)$	$(-2,-2,-2)$
$N_E^c(2,2,2)$	$(1,5,0)$	$(1,-3,2)$	$(4,0,5)$	–	$(-2,2,-3)$
$n(-4,0,-5)$	–	–	$(-2,-2,-2)$	$(-2,2,-3)$	–

For simplicity we make two further assumptions. First, we let  $\nu_E$  pair up with  $N_E^c$  to obtain a large Dirac mass  $M_{34}$ . Second, we assume an approximate  $Z_2$  symmetry to suppress vacuum expectation values stemming from SO(10) 16-plets in comparison with those from SO(10) 10's or singlets. The mass matrix in the basis  $(\nu_e, N_e^c, \nu_E, N_E^c, n)$ , where we have used small letters to denote entries with weak isospin  $\Delta I = 1/2$  and large letters to denote entries with  $\Delta I = 0$ , is

$$\mathcal{M} = \begin{bmatrix} 0 & m_{12} & 0 & M_{14} & 0 \\ m_{12} & M_{22} & 0 & m_{24} & 0 \\ 0 & 0 & 0 & M_{34} & m_{35} \\ M_{14} & m_{24} & M_{34} & 0 & m_{45} \\ 0 & 0 & m_{35} & m_{45} & 0 \end{bmatrix}.$$

It is convenient to diagonalize this matrix with respect to the large entry  $M_{34}$ ,

leading to

$$\mathcal{M}' = \begin{bmatrix} 0 & m_{12} & M_{14}/\sqrt{2} & M_{14}/\sqrt{2} & 0 \\ m_{12} & M_{22} & m_{24}/\sqrt{2} & m_{24}/\sqrt{2} & 0 \\ M_{14}/\sqrt{2} & m_{24}/\sqrt{2} & M_{34} & 0 & (m_{35}+m_{45})/\sqrt{2} \\ M_{14}/\sqrt{2} & m_{24}/\sqrt{2} & 0 & -M_{34} & (m_{45}-m_{35})/\sqrt{2} \\ 0 & 0 & (m_{35}+m_{45})/\sqrt{2} & (m_{45}-m_{35})/\sqrt{2} & 0 \end{bmatrix}.$$

Now we can perturb about the three eigenvectors  $[0, 1, 0, 0, 0]^T$ ,  $[0, 0, 1, 0, 0]^T$ , and  $[0, 0, 0, 1, 0]^T$  corresponding to the large eigenvalues  $M_{22}$ ,  $M_{34}$ ,  $-M_{34}$ . For the small masses, the resulting  $2 \times 2$  mass matrix in the  $(\nu_e, n)$  basis is

$$\mathcal{S}_2 = \begin{bmatrix} -m_{12}^2/M_{22} & -M_{14}m_{35}/M_{34} \\ -M_{14}m_{35}/M_{34} & -2m_{35}m_{45}/M_{34} \end{bmatrix}.$$

We look for solutions with small mixing and  $m_n > m_\nu$ :

$$\nu = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}, \quad n = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}, \quad t \equiv \tan \theta,$$

so we seek a small- $t$  solution of a quadratic equation in  $t$ , which in its linearized form is

$$t \simeq \left( \frac{m_{12}^2 M_{34}}{M_{14} m_{35} M_{22}} - \frac{2m_{45}}{M_{14}} \right)^{-1}.$$

Barring accidental cancellations, after several steps we get  $m_n > m_\nu$  with small mixing if  $M_{14} \ll m_{45}$  and

$$\left| \frac{m_{35} m_{45} M_{22}}{M_{34} m_{12}^2} \right| > 1, \quad \frac{m_{45}}{M_{14}} \gg 1,$$

The smallness of  $M_{14}$  is curious but achievable via the approximate  $Z_2$  symmetry mentioned earlier.

The neutrino mass matrix can be related to those for charged fermions at a unification scale. Thus,  $m_{12}$  and  $m_{35}$  are related to masses of quarks of charge  $2/3$ , while  $m_{24}$ ,  $m_{45}$ ,  $M_{14}$ , and  $M_{34}$  are related to charge  $-1/3$  quark and charged lepton masses. Specifically, for up-type quarks, the U(1) charges of masses are  $(-2, -2, -2)$ , corresponding to  $m_{21}$  and  $m_{35}$ . The relation of  $m_{12}$  to  $m_u$  is familiar from SO(10) unification. For down-type quarks and charged leptons, the correspondences are  $(-2, 2, -3) \sim m_{45}$ ,  $(1, 5, 0) \sim M_{14}$ ,  $(1, -3, 2) \sim m_{24}$ , and  $(4, 0, 5) \sim M_{34}$ . In the absence of mixing,  $m_{45}$  is related to Dirac masses of  $d$  and  $e$ , while  $M_{34}$  is related to Dirac masses of quarks and charged leptons in the 10 of SO(10). Weak universality suggests  $|m_{24}| \ll |m_{45}|$  (because isosinglet impurities in left-handed charged leptons and down-type quarks should be small), while there is less of a constraint on  $M_{14}$  as it has  $\Delta I = 0$ .

#### 4. Relevance for short-baseline neutrino oscillation experiments

The present model has mixing only within single families. In order to explain the LSND and MiniBooNE electron appearance signals one needs both muon and electron neutrinos to mix with the same sterile neutrino. The freedom of setting a sterile neutrino mass and mixing for one family (the matrix  $\mathcal{S}_2$  in the previous Section) is encouraging for the case of three families (which must be represented by a  $6 \times 6$  matrix). Furthermore, if data improve to the extent that three sterile neutrinos are needed to explain oscillations,  $E_6$  is available.

#### 5. One neutrino as a possible dark matter candidate

Another possible use of a third sterile  $\nu$  is as a warm dark matter candidate at the keV scale, as suggested some time ago.<sup>37,38</sup> For more recent reviews see Refs.<sup>23,39</sup> In contrast to many schemes, the present one distinguishes between right-hand neutrinos (usually taken very heavy, at the seesaw scale) and the  $n$ 's (one of which can easily have keV-scale mass).

There have been two claims for observation of an X-ray line near 3.5 keV.<sup>40,41</sup> These signals could arise from a 7 keV “neutrino” decaying to a photon and a much lighter “neutrino.” A corresponding signal is *not* seen, however, in the Milky Way.<sup>42</sup>

There are some special features of  $E_6$  concerning a 7 keV dark matter candidate. The Higgs vacuum expectation values considered here correspond to the five neutral complex scalar bosons in the  $27^*$  representation of  $E_6$ . The masses of these bosons are free parameters; two of the five are those of the minimal supersymmetric standard model or  $SO(10)$ . Exchanges of these bosons can produce the states  $n$ ; for example, in the processes

$$d_l + h_L^c \rightarrow n_L + N_{EL}^c \quad ; \quad e^- + E_L^+ \rightarrow n_L + N_{EL}^c .$$

A TeV-scale  $Z_N$  produced in the early universe would have appreciable branching ratio into  $nn^c$  pairs, so  $n$  are candidates for early overproduction unless their abundance is diluted by subsequent entropy production.<sup>43</sup>

#### 6. Summary

None of the various hints of sterile neutrinos rises to the level of a conclusive observation so it is crucial to strengthen or refute them. Some effects may be due to interesting non- $\nu$  physics: for example, if the MiniBooNE low-energy signal is photons and not electrons.

The grand unified group  $E_6$  [the next step up from  $SO(10)$ ] naturally incorporates three candidates for neutrinos with neither left-handed nor right-handed weak isospin.  $E_6$  breaking to the standard model times a particular  $U(1)_N$  allows a large Majorana mass for the right-handed neutrino  $N$  and hence the standard seesaw mechanism can proceed without constraints.<sup>30,44</sup> Masses and mixings of three sterile neutrinos are at one's disposal to fit oscillation data, assuming present anomalies are really due to sterile  $\nu$  and not something else.

If at most two sterile neutrinos are needed to fit anomalies successfully, a third is left over as a dark matter candidate. The  $E_6$  scheme appears to have enough free parameters to allow such a scenario to successfully navigate a number of constraints.

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